THEORY OF THE TEMPERATURE RELAXATION OF AN ELECTRON-ION PLASMA IN A HIGH-FREQUENCY ELECTRIC AND CONSTANT MAGNETIC FIELD

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The effective collision frequency of electrons and ions which leads to temperature equalization in a plasma in a constant magnetic field and a weak high-frequency electric field when the gyroscopic radius of the electrons is less than the Debye screening radius is determined. The corresponding values of the relaxation time are determined over a wide range of values of the ratio between the electron and ion temperatures, over a wide range of values of the magnetic and electric fields, and also as a function of the frequency of the external electric field.

Temperature equalization of the electrons and ions of a plasma in a high constant magnetic field when the Debye screening radius is greater than the gyroscopic radius of the particles has been investigated in [1, 2]. On the basis of these investigations a kinetic equation with integral collisions was proposed which takes into account the effect of the magnetic field on the motion of the colliding particles [3]. In this paper we consider the problem of the effect of a high-frequency electric field on the temperature relaxation time of a magnetized plasma.

As shown in [4], it is possible for growing oscillations to build up in a magnetized plasma in a strong high-frequency electric field. Hence, electric fields in which the drift velocity of the particles becomes greater than their thermal velocity are not considered below.

The investigation is made over a wide range of frequencies of the external electric field $\omega_0 > \nu_{ei}$ (ν_{ei} is the electron-ion collision frequency, an expression for which will be obtained below). In the low-frequency region $\omega_0 < \nu_{ei}$ the drift velocity of the electrons along the magnetic field will be determined not by the frequency ω_0 but by the electron collision frequency, which leads to another expression for the conductivity along the magnetic field.

Finally, it is shown that the collision time for collisions which occur due to Coulomb interaction, obtained in [2], still holds when the plasma is situated in an external high-frequency electric field and over a certain range of impact parameters is decisive.

1. The basis of our analysis is the kinetic equation (see [5])

$$\begin{gathered}\frac{\partial f_a}{\partial t}+e_a\left[\mathbf{E}(t)+\frac{1}{c}\mathbf{v}_a\times\mathbf{B}\right]\frac{\partial f_a}{\partial \mathbf{p}_a}\\ =\sum_b\frac{\partial}{\partial p_a{}^k}\iint\int_{-\infty}^{0}\frac{\partial U_{ab}(|\mathbf{r}_a-\mathbf{r}_b|)}{\partial r_a{}^k}\frac{\partial U_{ab}(|\mathbf{R}_a-\mathbf{R}_b|)}{\partial R_a{}^j}\\ \times\left(\frac{\partial}{\partial P_a{}^j}-\frac{\partial}{\partial P_b{}^j}\right)f_a(\mathbf{P}_a,t+\tau)f_b(\mathbf{P}_b,t+\tau)\,d^3p_b\,d^3r_b\,d\tau\end{gathered}\tag{1.1}$$

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Here f_a is the distribution function of particles of type a, U_{ab} is the screening potential of Coulomb interaction, and $\mathbf{P}_a$ and $\mathbf{R}_a$ are the momentum and coordinate at the instant of time $t+\tau$ of the particle a, which undergoes Coulomb interaction with the particle b, if at the instant of time t its momentum and coordinate have the values $\mathbf{p}_a$ and $\mathbf{r}_a$.

In order to obtain the limit of the collision time due to Coulomb interaction in accordance with [2], we use for $\mathbf{P}_a$ and $\mathbf{R}_a$ not the zeroth but the first approximation to the Coulomb interaction:

$$\mathbf{R}_a(t+\tau, \mathbf{r}_a, \mathbf{p}_a, \mathbf{r}_b, \mathbf{p}_b) = \mathbf{r}_a + \frac{1}{m_a}\int_t^{t+\tau} \mathbf{P}_a(t')\,dt'$$

$$\mathbf{R}_a \equiv \mathbf{R}_a^{(0)} + \mathbf{R}_a^{(1)}, \qquad \mathbf{P}_a \equiv \mathbf{P}_a^{(0)} + \mathbf{P}_a^{(1)}$$

$$\mathbf{P}_a^{(0)}(t+\tau, \mathbf{r}_a, \mathbf{p}_a, \mathbf{r}_b, \mathbf{p}_b) = \mathbf{b}(\mathbf{b}\cdot\mathbf{p}_a) - \mathbf{b}\times\mathbf{p}_a \sin\Omega_a\tau - \mathbf{b}\times(\mathbf{b}\times\mathbf{p}_a)\cos\Omega_a\tau$$

$$+ e_a \int_t^{t+\tau} [\mathbf{b}(\mathbf{b}\cdot\mathbf{E}(t')) - \mathbf{b}\times\mathbf{E}(t')\sin\Omega_a(t+\tau-t') - \mathbf{b}\times(\mathbf{b}\times\mathbf{E}(t'))\cos\Omega_a(t+\tau-t')]\,dt'$$

$$\mathbf{P}_a^{(1)}(t+\tau, \mathbf{r}_a, \mathbf{p}_a, \mathbf{r}_b, \mathbf{p}_b) = \int_t^{t+\tau} [\mathbf{b}(\mathbf{b}\cdot\mathbf{f}_{ab}) - \mathbf{b}\times\mathbf{f}_{ab}\sin\Omega_a(t+\tau-t') - \mathbf{b}\times(\mathbf{b}\times\mathbf{f}_{ab})\cos\Omega_a(t+\tau-t')]\,dt'$$

$$\mathbf{f}_{ab}(t) = -\frac{\partial}{\partial\mathbf{R}_a^{(0)}(t)} U_{ab}(|\mathbf{R}_a^{(0)} - \mathbf{R}_b^{(0)}|) \quad \Omega_a = \frac{e_a B}{m_a c}, \quad \mathbf{b} = \frac{\mathbf{B}}{B}$$

Since momentum relaxation occurs much more rapidly than temperature relaxation, we can take as the distribution functions the Maxwell functions of argument

$$\mathbf{p}_a - e_a \int_{-\infty}^{t} [\mathbf{b}(\mathbf{b}\cdot\mathbf{E}(t')) - \mathbf{b}\times\mathbf{E}(t')\sin\Omega_a(t-t') - \mathbf{b}\times(\mathbf{b}\times\mathbf{E}(t'))\cos\Omega_a(t-t')]\,dt'$$

Multiplying Eq. (1.1) by

$$\frac{1}{2m_a}\left\{\mathbf{p}_a - e_a \int_{-\infty}^{t} [\mathbf{b}(\mathbf{b}\cdot\mathbf{E}(t')) - \mathbf{b}\times\mathbf{E}(t')\sin\Omega_a(t-t') - \mathbf{b}\times(\mathbf{b}\times\mathbf{E}(t'))\cos\Omega_a(t-t')]\,dt'\right\}^2$$

and integrating over the momenta of the particles a, we obtain the temperature relaxation equation.

Before taking the integrals on the right side of the equation, we will discuss the effect of Coulomb interaction on the motion of the particles. Corrections of the first approximation of the Coulomb interaction to $\mathbf{R}_a$ were obtained as in [2], so that the characteristic collision time which arises due to taking into account the effect of Coulomb interaction

$$\tau_{\max}(k) \approx k^{-3/2}\left(\frac{m_e}{|ee_i|}\right)^{1/2}$$

obtained in [2], is unchanged. Hence, the integral with respect to τ in (1.1) must be taken from $-\tau_{\max}(k)$ to 0. Henceforth we can neglect the small additive corrections due to the effect of the Coulomb field on the momenta of the colliding particles. This is justified for the assumed logarithmic accuracy of the calculations.

To obtain the final expression we must now take the integrals on the right side of the temperature relaxation equation. We will take the time dependence of the electric field in the form

$$\mathbf{E}(t) = \mathbf{E}_0 \cos\omega_0 t$$

Proceeding as, for example, in [5], this equation can be written in the form

$$\frac{\partial T_a}{\partial t}=\sum_b (T_b-T_a)\frac{16e_a^2e_b^2n_b}{3m_am_b}\int\limits_{k_{\min}}^{k_{\max}}\int\limits_0^{\tau_{\max}(k)}\int\limits_0^{1/2\pi} k^2\tau^2\sin\theta\exp(-X_a-X_b)$$
$$\times\left(\cos^2\theta+\sin^2\theta\frac{\sin\Omega_a\tau}{\Omega_a\tau}\right)\left(\cos^2\theta+\sin^2\theta\frac{\sin\Omega_b\tau}{\Omega_b\tau}\right)\cos(k\rho_\parallel\cos\theta)J_0(k\rho_\perp\sin\theta)\,dk\,d\tau\,d\theta$$
$$+\sum_b\frac{16e_a^2e_b^2n_b}{3m_a}\int\limits_{k_{\min}}^{k_{\max}}\int\limits_0^{\tau_{\max}(k)}\int\limits_0^{1/2\pi}k\tau\sin\theta\exp(-X_a-X_b)\left(\cos^2\theta+\sin^2\theta\frac{\sin\Omega_a\tau}{\Omega_a\tau}\right) \tag{1.2}$$
$$\times\left[\rho_\parallel^{\bullet}\cos\theta\sin(k\rho_\parallel\cos\theta)J_0(k\rho_\perp\sin\theta)+\frac{(\rho\times\mathbf{b})\cdot(\rho^{\bullet}\times\mathbf{b})}{\rho_\perp}\sin\theta\cos(k\rho_\parallel\cos\theta)J_1(k\rho_\perp\sin\theta)\,dk\,d\tau\,d\theta\right]$$

where T is the temperature measured in energy units, $k_{\min}=r_D^{-1}$, $k_{\max}=r_{\min}^{-1}$, as in the usual Landau collision integral, θ is the angle between the vectors $\mathbf{b}$ and $\mathbf{k}$, and

$$\rho=\rho_a-\rho_b,\quad \rho^{\bullet}=\frac{\partial}{\partial\tau}\rho,\quad \rho_\parallel=\mathbf{b}\cdot\rho,\quad \rho_\perp=|\mathbf{b}\times\rho|$$
$$\rho_a(\tau)=2\sin\frac{\omega_0\tau}{2}\left[\frac{e_a}{m_a\omega_0^2}\mathbf{b}(\mathbf{b}\cdot\mathbf{E}_0)\sin\omega_0\left(t-\frac{\tau}{2}\right)\right.$$
$$+\frac{e_a\Omega_a}{m_a\omega_0(\omega_0^2-\Omega_a^2)}\mathbf{b}\times\mathbf{E}_0\cos\omega_0\left(t-\frac{\tau}{2}\right)$$
$$\left.+\frac{e_a}{m_a(\omega_0^2-\Omega_a^2)}\mathbf{b}\times(\mathbf{E}_0\times\mathbf{b})\sin\omega_0\left(t-\frac{\tau}{2}\right)\right]$$
$$X_a=\frac{1}{2}k^2v_{Ta}^2\tau^2\left[\cos^2\theta+\frac{4}{\Omega_a^2\tau^2}\sin^2\theta\sin^2\frac{\Omega_a\tau}{2}\right]$$
$$vT_a^2=T_a/m_a$$

J_n is a Bessel function of order n.

We will average the expression obtained over an oscillation period of the external field. As can be seen from (1.2), due to oscillations of the Bessel and trigonometric functions in the regions

$$k\rho_\parallel\cos\theta>1,\qquad k\rho_\perp\sin\theta>1$$

the integral is small, so that integration can be carried out over the regions where

$$k\rho_\parallel\cos\theta\lesssim 1,\qquad k\rho_\perp\sin\theta\lesssim 1$$

In these regions we can expand the Bessel and trigonometric functions in series, and confining ourselves to the first terms of the expansion, we can carry out the above averaging. As a result, if the plasma consists of electrons and one type of ion, we obtain the following expression:

$$\frac{\partial T_e}{\partial t}=(T_i-T_e)\frac{2m_e}{m_i}\nu_0\left(\frac{2}{\pi}\right)^{1/2}\int\limits_{\varkappa_{\min}}^{\varkappa_{\max}}\int\limits_0^{t_{\max}(\varkappa)}\int\limits_0^1\varkappa^2t^2\exp\left[-\frac{\varkappa^2t^2}{2}\psi(^1/_2t)\right]\exp\left\{-\frac{\varkappa^2t^2x^2}{2}\left[1+\frac{v_{Ti}^2}{v_{Te}^2}-\psi(^1/_2t)\right]\right\}$$
$$\times\left[x^2\left(1-\frac{\sin t}{t}\right)+\frac{\sin t}{t}\right]\left[x^2\left(1-\frac{\Omega_e}{\Omega_it}\sin\frac{\Omega_it}{\Omega_e}\right)+\frac{\Omega_e}{\Omega_it}\sin\frac{\Omega_it}{\Omega_e}\right]d\varkappa\,dt\,dx$$
$$+\frac{e^2\nu_0}{m_e\omega_0^2}\left(\frac{2}{\pi}\right)^{1/2}\int\limits_{\varkappa_{\min}}^{\varkappa_{\max}}\int\limits_0^{t_{\max}(\varkappa)}\int\limits_0^1\varkappa^2\frac{\Omega_e}{\omega_0}t\sin\frac{\omega_0}{\Omega_e}t\exp\left[-\frac{\varkappa^2t^2}{2}\psi(^1/_2t)\right] \tag{1.3}$$
$$\times\exp\left\{-\frac{\varkappa^2t^2x^2}{2}\left[1+\frac{v_{Ti}^2}{v_{Te}^2}-\psi(^1/_2t)\right]\right\}\left[x^2\left(1-\frac{\sin t}{t}\right)\right.$$
$$\left.+\frac{\sin t}{t}\right]\left[x^2E_\parallel^2+\frac{1-x^2}{2}Z^2(\omega_0)E_\perp^2\right]d\varkappa\,dt\,dx$$

Integration with respect to $\varkappa$ and t is only carried out in those regions where

$$2\frac{\Omega_e}{\omega_0}\varkappa\sin\frac{\omega_0t}{2\Omega_e}<\frac{v_{Te}}{v_{E\parallel(\perp)}} \tag{1.4}$$

Here

$$\nu_0 = \frac{4}{3}\frac{\sqrt{2\pi}\, e^2 e_i^2 n_i}{\sqrt{m_e}\, T_e^{3/2}}$$

$$\varkappa_{\min} = \frac{\rho_e}{r_D}, \qquad \varkappa_{\max} = \frac{\rho_e}{r_{\min}}, \qquad \rho_a = \frac{v_{Ta}}{\Omega_a}$$

$$t_{\max}(\varkappa) = \Omega_e \tau_{\max}(k) = \varkappa_{\max}^{1/2} \varkappa^{-3/2}$$

$$\psi(t) = \frac{1}{t^2}\left(\sin^2 t + \frac{\rho_i^2}{\rho_e^2}\sin^2\frac{\Omega_i t}{\Omega_e}\right)$$

$$Z(\omega_0) = \frac{\omega_0}{\Omega_e}\left[\left(\frac{\Omega_e^2}{\omega_0^2 - \Omega_e^2} + \frac{\Omega_i^2}{\omega_0^2 - \Omega_i^2}\right)^2 + \left(\frac{\omega_0 \Omega_e}{\omega_0^2 - \Omega_e^2} + \frac{\omega_0 \Omega_i}{\omega_0^2 - \Omega_i^2}\right)^2\right]^{1/2}$$

$$E_\parallel = \mathbf{b}\cdot\mathbf{E}_0, \qquad E_\perp = |\mathbf{b}\times\mathbf{E}_0|$$

$$\mathbf{v}_E = \mathbf{v}_{Ee} - \mathbf{v}_{Ei}, \quad v_{E\parallel} = \mathbf{b}\cdot\mathbf{v}_E, \quad v_{E\perp} = |\mathbf{b}\times\mathbf{v}_E|$$

Here $\mathbf{v}_E$ is the drift velocity of the particles in the electric field. The quantities $v_{E\parallel}$ and $v_{E\perp}$ can be obtained from the equations of motion of the particles in electric and magnetic fields:

$$v_{E\parallel} \approx \frac{eE_\parallel}{m_e \omega_0}, \qquad v_{E\perp} \approx \frac{eE_\perp}{m_e \omega_0} Z(\omega_0)$$

2. Bearing in mind the large logarithmic expressions, we will only retain the leading terms, i.e., we will neglect quantities of the order of unity in comparison with the logarithmic and double logarithmic expressions. We will write (1.3) in the form

$$\frac{\partial T_e}{\partial t} = \frac{T_i - T_e}{\tau_T} + \sigma_{jk} E_j E_k$$
$$\tau_T^{-1} = \frac{2m_e}{m_i}\nu_0 (L_1 + \delta L_1) \tag{2.1}$$

τ_T is the temperature relaxation time, σ_{jk} is the conductivity tensor,

$$L_1 = \left(\frac{2}{\pi}\right)^{1/2} \int_{\varkappa_{\min}}^{\varkappa_{\max}} \int_0^1 \varkappa^2 t^2 \exp\frac{-\varkappa^2 t^2}{2}\, d\varkappa\, dt \tag{2.2}$$

$$\delta L_1 = \int_{\varkappa_{\min}}^{1} \int_{1/\varkappa} \frac{d\varkappa\, dt}{\varkappa t} \tag{2.3}$$

where the upper limit of integration with respect to t is the least of the following three expressions:

$$\varkappa_{\max}^{1/2}\varkappa^{-3/2}, \qquad \frac{v_{Te}}{\varkappa v_{Ti}}, \qquad \frac{\Omega_e}{\Omega_i}$$

In expressions (2.2) and (2.3) integration is carried out only in those regions of variation of $\varkappa$ and t where the following relation holds:

$$2\frac{\Omega_e}{\omega_0}\varkappa \sin\frac{\omega_0 t}{2\Omega_e} < \frac{v_{Te}}{v_E}, \quad v_E = \max\{v_{E\parallel}, v_{E\perp}\}, \tag{2.4}$$

We will consider only the first term of expression (2.1) proportional to the temperature difference. The second term is related to the heat dissipated in the plasma due to the action of the electric field **E**. Hence, the conductivity tensor σ_{jk} agrees with that calculated in [5], where the conductivity of a magnetized plasma in a weak high-frequency electric field was considered.

The formal difference between expressions (2.2) and (2.3) and the corresponding expressions given in [2], where temperature relaxation in a magnetized plasma was considered in the absence of an electric field, consists of the presence of the additional limitation (2.4), which takes into account the upper limit of the interaction time due to particle drift in the electric field. Note that this limitation is important for a frequency of the electric field $\omega_0 < \Omega_e$ and $v_E > v_{Ti}$.

For an unmagnetized plasma ($\rho_e > r_D$) we obtain

$$L_1 = \ln (r_D/r_{\min})$$

For a magnetized plasma ($\rho_e < r_D$)

$$L_1 = \ln (\rho_e / r_{\min})$$

In the first case no double logarithmic expressions occur; in the second case double logarithmic corrections occur due to (2.3). As stated above, in the regions

$$\omega_0 > \Omega_e, \quad v_E > v_{Ti}$$

the effect of the electric field is negligble, so that we have the expressions obtained in [2]. In the opposite case we have the new expressions obtained here.

To simplify the equations we will introduce the following notation:

$$r_0 = r_{\min} v_{Te}^2 / v_{Ti}^2$$
$$l_1 = \ln \frac{r_D}{\rho_e}, \quad l_2 = \ln \frac{v_E}{\Omega_i \rho_e}, \quad l_3 = \ln \frac{v_E}{\omega_0 \rho_e}$$
$$l_4 = \ln \frac{v_{Te}}{v_E}, \quad l_5 = \ln \frac{v_{Te}}{v_{Ti}}, \quad l_6 = \frac{1}{2} \ln \frac{r_0 v_{Ti}^2}{\rho_e v_E^2}$$
$$l_7 = \ln \frac{\rho_i}{\rho_e}, \quad l_8 = \frac{1}{2} \ln \frac{\omega_0 r_0}{v_E}, \quad l_9 = \frac{1}{6} \ln \frac{r_0 \rho_i^2 \omega_0^3}{v_E^3}$$

It should be noted that the first six logarithms are linearly independent while the remaining three can be expressed linearly in terms of the first six.

Using the above notation, we can write the expressions obtained for δL_1 as follows:

In the regions $\nu_{ei} < \omega_0 < \Omega_i$ and $\Omega_i < \omega_0 < \Omega_e$

$$\begin{gathered} \delta L_1 = l_1 l_4 \\ v_E / \omega_0, \ v_E / \Omega_i > r_D > \rho_e > r_0 v_{Ti}^2 / v_E^2, \quad v_{Te} > v_E > v_{Ti} \\ \delta L_1 = l_1 l_4 - l_6^2 \\ v_{Te} / \Omega_i > r_D > r_0 v_{Ti}^2 / v_E^2 > \rho_e, \quad v_{Te} > v_E > v_{Ti} \end{gathered} \tag{2.5}$$

In the low-frequency region $\nu_{ei} < \omega_0 < \Omega_i$ in addition to (2.5) we have

$$\begin{gathered} \delta L_1 = l_1 l_4 - {}^1/_2 \ (l_1 - l_2)^2 \\ v_{Te} / \Omega_i > r_D > v_E / \Omega_i > \rho_i > \rho_e > r_0 v_{Ti}^2 / v_E^2 \\ \delta L_1 = l_1 l_4 - {}^1/_2 (l_1 - l_2)^2 - l_6^2 \\ v_{Te} / \Omega_i > r_D > v_E / \Omega_i > \rho_i, \ r_0 v_{Ti}^2 / v_E^2 > \rho_e \\ \delta L_1 = l_2 l_4 + {}^1/_2 \, l_4^2 \\ r_D > v_{Te} / \Omega_i > v_E / \Omega_i > \rho_i > \rho_e > r_0 v_{Ti}^2 / v_E^2 \\ \delta L_1 = l_2 l_4 + {}^1/_2 l_4^2 - l_6^2 \\ r_D > v_{Te} / \Omega_i > v_E / \Omega_i > \rho_i, \ r_0 v_{Ti}^2 / v_E^2 > \rho_e \end{gathered} \tag{2.6}$$

In the low-frequency region $\nu_{ei} < \omega_0 < \Omega_i$ no double logarithmic expressions occur.

In the frequency region $\Omega_i < \omega_0 < \Omega_e$ in addition to (2.5) we also have the following expressions:

$$\begin{gathered} \delta L_1 = l_3 l_4 + (l_1 - l_3) \, l_5 \\ \rho_i > r_D > v_E / \omega_0 > r_0, \ \rho_e > r_0 v_{Ti}^2 / v_E^2 \\ \delta L_1 = l_3 l_4 + (l_1 - l_3) \, l_5 - {}^1/_2 \, (l_1 - l_7)^2 \\ v_{Te} / \Omega_i > r_D > \rho_i > v_E / \omega_0 > r_0, \ \rho_e > r_0 \, v_{Ti}^2 / v_E^2 \\ \delta L_1 = l_3 l_4 + (l_1 - l_3) \, (l_5 - l_7) - {}^1/_2 \, l_1^2 + {}^1/_2 \, l_3^2 \\ v_{Te} / \Omega_i > r_D > v_E / \omega_0 > \rho_i, \ r_0^{1/3} \rho_i^{2/3}, \ \rho_e > r_0 v_{Ti}^2 / v_E^2 \\ \delta L_1 = l_3 l_4 + (l_7 - l_3) \, l_5 + {}^1/_2 \, l_5^2 \\ r_D > v_{Te} / \Omega_i > \rho_i > v_E / \omega_0 > r_0, \ \rho_e > r_0 v_{Ti}^2 / v_E^2 \\ \delta L_1 = l_3 l_4 + {}^1/_2 \, (l_5 + l_7 - l_3)^2 \\ r_D > v_{Te} / \Omega_i > v_E / \omega_0 > \rho_i, \ r_0^{1/3} \rho_i^{2/3}, \ \rho_e > r_0 v_{Ti}^2 / v_E^2 \end{gathered} \tag{2.7}$$

$$
\begin{gathered}
\delta L_1 = l_3 l_4 + (l_1 - l_3)\, l_5 - {}^1/_2\, (l_1 - l_7)^2 - l_8{}^2 \\
v_{Te} / \Omega_i > r_D > \rho_i > r_0 > v_E / \omega_0 > \rho_e > r_0 v_{Ti}{}^2 / v_E{}^2 \\
\delta L_1 = l_3 l_4 + (l_1 - l_3)\, (l_5 - l_7) - {}^1/_2\, l_1{}^2 + {}^1/_2\, l_3{}^2 - 3 l_9{}^2 \\
v_{Te} / \Omega_i > r_D > r_0{}^{1/3} \rho_i{}^{2/3} > v_E / \omega_0,\ \rho_i > \rho_e > r_0 v_{Ti}{}^2 / v_E{}^2 \\
\delta L_1 = l_3 l_4 + (l_1 - l_3)\, (l_5 - l_8) + {}^1/_4\, (l_1 - l_3)^2 \\
r_0,\ r_0{}^{1/3} \rho_i{}^{2/3} > r_D > v_E / \omega_0 > \rho_e > r_0 v_{Ti}{}^2 / v_E{}^2 \\
\delta L_1 = l_3 l_4 + (l_1 - l_3)\ l_5 - l_8{}^2 \\
\rho_i > r_D > r_0 > v_E / \omega_0 > \rho_e > r_0 v_{Ti}{}^2 / v_E{}^2 \\
\delta L_1 = l_3 l_4 + (l_7 - l_3)\, l_5 + {}^1/_2\, l_5{}^2 - l_8{}^2 \\
r_D > v_{Te} / \Omega_i > \rho_i > r_0 > v_E / \omega_0 > \rho_e > r_0 v_{Ti}{}^2 / v_E{}^2 \\
\delta L_1 = l_3 l_4 + {}^1/_2\, (l_5 + l_7 - l_3)^2 - 3 l_9{}^2 \\
r_D > v_{Te} / \Omega_i > r_0{}^{1/3} \rho_i{}^{2/3} > \rho_i,\quad v_E / \omega_0 > \rho_e > r_0 v_{Ti}{}^2 / v_E{}^2
\end{gathered}
$$

Common to the expressions (2.7) is the fact that in the region of the impact parameters $v_E/\omega_0 > r > \rho_e$ the interaction time is limited by particle drift in the electric field. This is due to the fact that in (2.7) in all the expressions $\rho_e > r_0 v_{Ti}{}^2/v_E{}^2$. In the opposite case we have

$$
\begin{gathered}
\delta L_1 = l_3 l_4 - l_6{}^2 + (l_1 - l_3) l_5 \\
\rho_i > r_D > v_E / \omega_0 > r_0 > r_0 v_{Ti}{}^2 / v_E{}^2 > \rho_e
\end{gathered}
\tag{2.8}
$$

$$
\begin{gathered}
\delta L_1 = l_3 l_4 - l_6{}^2 + (l_1 - l_3)\ l_5 - {}^1/_2\, (l_1 - l_7)^2 \\
v_{Te} / \Omega_i > r_D > \rho_i > v_E / \omega_0 > r_0 > r_0 v_{Ti}{}^2 / v_E{}^2 > \rho_e \\
\delta L_1 = l_3 l_4 - l_6{}^2 + (l_1 - l_3)\, (l_5 - l_7) - {}^1/_2\, l_1{}^2 + {}^1/_2\, l_3{}^2 \\
v_{Te} / \Omega_i > r_D > v_E / \omega_0 > \rho_i,\ r_0{}^{1/3} \rho_i{}^{2/3},\ r_0 v_{Ti}{}^2 / v_E{}^2 > \rho_e \\
\delta L_1 = l_3 l_4 - l_6{}^2 + (l_7 - l_3)\ l_5 + {}^1/_2\, l_5{}^2 \\
r_D > v_{Te} / \Omega_i > \rho_i > v_E / \omega_0 > r_0 > r_0 v_{Ti}{}^2 / v_E{}^2 > \rho_e \\
\delta L_1 = l_3 l_4 - l_6{}^2 + {}^1/_2\, (l_5 + l_7 - l_3)^2 \\
r_D > v_{Te} / \Omega_i > v_E / \omega_0 > \rho_i,\ r_0{}^{1/3} \rho_i{}^{2/3},\ r_0 v_{Ti}{}^2 / v_E{}^2 > \rho_e \\
\delta L_1 = l_3 l_4 - l_6{}^2 + (l_1 - l_3)\ l_5 - {}^1/_2\, (l_1 - l_7)^2 - l_8{}^2 \\
v_{Te} / \Omega_i > r_D > \rho_i > r_0 > v_E / \omega_0 > r_0 v_{Ti}{}^2 / v_E{}^2 > \rho_e \\
\delta L_1 = l_3 l_4 - l_6{}^2 + (l_1 - l_3)\ (l_5 - l_7) - {}^1/_2\, l_1{}^2 + {}^1/_2\, l_3{}^2 - 3 l_9{}^2 \\
v_{Te} / \Omega_i > r_D > r_0{}^{1/3} \rho_i{}^{2/3} > v_E / \omega_0 > r_0 v_{Ti}{}^2 / v_E{}^2 > \rho_e, \qquad r_0 > \rho_i \\
\delta L_1 = l_3 l_4 - l_6{}^2 + (l_1 - l_3)\ (l_5 - l_8) + {}^1/_4\, (l_1 - l_3)^2 \\
r_0,\ r_0{}^{1/3} \rho_i{}^{2/3} > r_D > v_E / \omega_0 > r_0 v_{Ti}{}^2 / v_E{}^2 > \rho_e \\
\delta L_1 = l_3 l_4 - l_6{}^2 + (l_1 - l_3)\ l_5 - l_8{}^2 \\
\rho_i > r_D > r_0 > v_E / \omega_0 > r_0\, v_{Ti}{}^2 / v_E{}^2 > \rho_e \\
\delta L_1 = l_3 l_4 - l_6{}^2 + (l_7 - l_3)\ l_5 + {}^1/_2\, l_5{}^2 - l_8{}^2 \\
r_D > v_{Te} / \Omega_i > \rho_i > r_0 > v_E / \omega_0 > r_0 v_{Ti}{}^2 / v_E{}^2 > \rho_e \\
\delta L_1 = l_3 l_4 - l_6{}^2 + {}^1/_2\, (l_5 + l_7 - l_3)^2 - 3 l_9{}^2 \\
r_D > v_{Te} / \Omega_i > r_0{}^{1/3} \rho_i{}^{2/3} > v_E / \omega_0 > r_0 v_{Ti}{}^2 / v_E{}^2 > \rho_e, \qquad r_0 > \rho_i
\end{gathered}
$$

Common to expressions (2.8) is the fact that in the range of impact parameters

$$r_0 v_{Ti}{}^2 / v_E{}^2 > r > \rho_e$$

the interaction time is limited by the effect of Coulomb acceleration of the particles, and in the range of impact parameters

$$v_E / \omega_0 > r > r_0\, v_{Ti}{}^2 / v_E{}^2$$

the interaction time is limited by particle drift in the electric field.

3. The results obtained hold, as stated above, for values of the electric field for which the particle drift velocity v_E is less than the thermal velocity of the electrons. However, it follows from results obtained by Andreev and Kiriy, kindly communicated to the authors prior to publication, that when $v_E < v_{Te}$, parametric resonance is possible if the frequency of the electric field is close to the frequencies of electron plasma oscillations in a constant magnetic field:

$$
\begin{gathered}
(\omega_{1,2})^2 = {}^1/_2\ \{(\omega_{Le}{}^2 + \Omega_e{}^2) \pm [(\omega_{Le}{}^2 + \Omega_e{}^2)^2 - 4\omega_{Le}{}^2 \Omega_e{}^2\ \cos^2\theta]^{1/2}\} \\
\omega_{Le} = (4\pi\, e^2 n_e / m_e)^{1/2}
\end{gathered}
$$

Here ω_{Le} is the Langmuir frequency of the electrons, and θ is the angle between the direction of propagation of the oscillations and the direction of the constant magnetic field **B**.

In a magnetized plasma the upper branch lies within the limits

$$\Omega_e \leqslant \omega_1 \leqslant (\Omega_e^2 + \omega_{Le}^2)^{1/2}$$

while the lower branch lies within the limits

$$\Omega_e [(\omega_{Li}^2 + \Omega_i^2) / (\omega_{Le}^2 + \Omega_e^2)]^{1/2} \leqslant \omega_2 \leqslant \omega_{Le}$$

Hence, the results obtained must be used in the frequency ranges

$$\begin{gathered} \nu_{ei} < \omega_0 < \Omega_i, \quad \Omega_i < \omega_0 < \Omega_e \left(\frac{\omega_{Li}^2 + \Omega_i^2}{\omega_{Le}^2 + \Omega_e^2}\right)^{1/2} \\ \omega_{Le} < \omega_0 < \Omega_e, \quad (\Omega_e^2 + \omega_{Le}^2)^{1/2} < \omega_0 \end{gathered} \tag{3.1}$$

These are the regions in which parametric resonance is impossible. In the parametric-resonance regions, for example, in the lower branch ω_2, the results obtained only make sense if the electric field E_0 is less than the threshold field E_*, given by

$$\frac{E_*^2}{4\pi n_e T_e} = \sqrt{8\pi}\, \frac{\nu_{ei}\omega_{Li}}{\omega_{Le}^2} \frac{\Omega_e^2}{\Omega_e^2 \cos^2\chi + \omega_{Le}^2 \sin^2\chi} |\cos\theta(\omega_0)|$$

where χ is the angle between the directions of the vectors **B** and $\mathbf{E}_0$, while $\theta(\omega_0)$ is given by the expression

$$\cos^2\theta(\omega_0) = \frac{\omega_0^2 (\omega_{Le}^2 + \Omega_e^2 - \omega_0^2)}{\omega_{Le}^2 \Omega_e^2}$$

The expression derived holds in the long-wave limit ($\omega_s < \Omega_i$) in a magnetized plasma ($\omega_s = k\omega_{Li} r_D$ is the ion-acoustic oscillation frequency). More detailed expressions for the thresholds can be found in the above-mentioned paper after publication.

4. The above investigation shows that in a magnetized plasma in a high-frequency electric field double logarithmic corrections to the temperature relaxation time occur, which depend on the intensity and frequency of the electric field. This dependence was investigated with the following limitations:

1) The drift velocity of the particles in the electric field doe not exceed the thermal velocity of the electrons ($v_E < v_{Te}$); this is a limitation on the value of the electric field.

2) The cyclotron radius of the electrons should not be greater than that of the ions ($\rho_e < \rho_i$), and the thermal velocity of the ions should be less than the thermal velocity of the electrons ($v_{Ti} < v_{Te}$); this is a limitation on the ratio of the electron and ion temperatures, since the last two inequalities can be written as

$$\frac{m_i}{m_e} > \frac{T_i}{T_e} > z^2 \frac{m_e}{m_i}$$

(z is the charge on the ion in units of e)

3) The frequency of the electric field lies in the ranges which satisfy inequalities (3.1).

Note that under these conditions the dependence on the frequency and the electric field appears in the temperature relaxation time only when $\omega_0 < \Omega_e$. In the opposite case the electric field has no effect on the first term of Eq. (2.1).

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